Algebraic Groups and Representations
Lyon
June 2, 2014 to July 11, 2014

Conference: Structure of Algebraic Groups

June 23-27, 2014

Yves André (IMJ, Paris) — *Espaces quasi homogènes : aspects tannakiens et algébro-différentiels*

Aravind Asok (University of Southern California, Los Angeles) — *Vector bundles and $\mathbf{A}^1$-homotopy theory of SL_2*

I will discuss some results regarding the structure of the $\mathbf{A}^1$-homotopy sheaves of SL_2. I will explain how these results can be applied to better understand vector bundles of small rank on certain smooth affine quadric hypersurfaces. This talk is based on joint work with Jean Fasel.

Sanghoon Baek (KAIST, Daejeon) — *Torsion in the filtrations of a product of Brauer-Severi varieties*

We discuss the torsion subgroups in the topological and gamma filtrations of the Grothendieck group of a product of twisted projective spaces. In particular, we investigate a lower bound for the torsion in the filtrations of a variety associated to a division algebra and general bounds for the torsion of the 2nd quotient in the filtrations of the products of flag surfaces. We also show that in some cases the bound is sharp.

Baptiste Calmès (Université d'Artois, Lens) — *Oriented equivariant cohomology and restriction to fixed points*

Let G be a split semisimple linear algebraic group over a field k, let T be a maximal split torus, and let B be a Borel subgroup containing T. Our main object of study is the ring structure of $h_{\mathrm{T}}(\mathrm{G}/\mathrm{B})$, where h_{T} denotes a T-equivariant oriented cohomology theory on smooth projective varieties over k. It could be Chow groups or the Grothendieck group, or a much more complicated cohomology theory such as algebraic cobordism.

I will explain how the well-known method of restriction to fixed points used by various authors (e.g. Brion, Atiyah-Bott or Goretsky-Kottwitz-MacPherson) can be carried out in general for any equivariant oriented cohomology theory h, by making use of an algebraic formalism generalizing previous work of Kostant, Kumar and Demazure by taking as input the formal group law of h. I will emphasize the difference between the classical examples of Chow groups and K-theory, and the other cases, and I will also mention the parabolic case G/P. This is joint work with K. Zainoulline and C. Zhong.

Brian Conrad (Stanford University) — *Compactification of algebraic groups*

I will discuss joint work with Gille and Prasad on using the structure theory of pseudo-reductive groups to prove a general equivariant compactification theorem (announced earlier by Gabber) for smooth connected affine groups.

Charles De Clercq (Université Paris 13 nord, Villetaneuse) — *Motivic equivalence of semisimple algebraic groups*

Two semisimple algebraic groups are said to be motivic equivalent if the motives of the associated projective homogeneous varieties are isomorphic. We will show in the lecture how the motivic equivalence of two such groups can be characterized by some coloured Dynkin diagram, in the same fashion as Tits' classification of algebraic groups. We will thereafter discuss consequences of this classification on birational geometry of some projective homogeneous varieties, as well as its link with other classical invariants of semisimple algebraic groups.

Cyril Demarche (Université Pierre et Marie Curie, Paris) — *The fundamental group of a homogeneous space of a linear algebraic group*

If X is a homogeneous space of a connected linear algebraic group G over an algebraically closed field of characteristic p, we prove explicit formulas for its topological fundamental group (if $p = 0$) and its prime-to-p étale fundamental group (if $p > 0$). Those formulas are purely algebraic formulas, involving modules of characters of the group G and of the stabilizer of a geometric point of X, and they generalize recent results by Brion and Szamuely. This is joint work with Mikhail Borovoi.

Adrien Dubouloz (Univeristé de Bourgogne, Dijon) — *Affine surfaces with wild automorphism groups*

It is well known that a complete algebraic variety admits a largest connected algebraic group of automorphisms: the identity component of its group scheme of automorphisms, equipped with its reduced structure. In contrast, even though it always carries the weaker algebraic structure of a group object in the category of ind-schemes, the automorphism group of an affine variety is in general not a group scheme, and its identity component can be infinite dimensional. This phenomenon already occurs for simple affine surfaces such as the affine plane, and it is a natural exploratory question to decide how remote from algebraic groups the automorphism groups of affine surfaces can be. In this talk, I will review a collection of results in this direction obtained in collaboration with S. Lamy and J. Blanc via the study of birational relations between suitable projective models of these surfaces. One of the outcome is the existence of a moduli of surfaces with "huge" groups of automorphisms, for instance connected groups containing a normal subgroup which cannot be generated by any countable family of algebraic groups and whose quotients contain free groups over uncountable sets of generators.

Roman Fedorov (Kansas University) — *Principal bundles over affine lines*

Let U be a spectrum of a regular local ring, let G be a reductive group scheme over U. I will discuss principal G-bundles over the product of U and the affine line. I will

explain the relation with affine Grassmannians and provide examples of "exotic" principal bundles. That is, of non-trivial principal bundles becoming trivial after removing from $A^1 x U$ a subscheme etale over U.

Gunter Malle (Kaiserslautern Universität) — *Products and commutators of classes in algebraic groups*

We classify pairs of conjugacy classes in almost simple algebraic groups whose product consists of finitely many classes. This leads to several interesting families of examples which are related to a generalization of the Baer-Suzuki theorem for finite groups. We also answer a question of Pavel Shumyatsky on commutators of pairs of conjugacy classes in simple algebraic groups. It turns out that the resulting examples are exactly those for which the product also consists of only finitely many classes. This is joint work with R. Guralnick and P. Tiep.

Alexander Neshitov (University of Ottawa) — *Invariants of degree 3 and torsion in the Chow ring of a versal flag variety*

The talk is based on the joint project with Alexander Merkurjev and Kirill Zainoulline. We will discuss the connection between degree 3 cohomological invariants of a semisimple split group and the torsion in the Chow group of codimension two cycles of the corresponding versal flag variety. For a simple split group we prove that these two groups coincide. In general case we describe the difference in terms of the character lattice.

Arturo Pianzola (University of Alberta, Edmonton) — *The case for loop-reductive group schemes: Applications to infinite dimensional Lie theory and connections to Dessins d'enfants*

Gopal Prasad (Ann Arbor) — *Split spherical Tits systems in quasi-reductive groups*

Given a quasi-reductive group G over an infinite field k (for example, G a connected reductive k-group), we will indicate a proof of the rather surprising result that any split (or even "weakly-split") spherical Tits system in G(k) is standard (that is, with B = the group of k-rational points of a pseudo-parabolic k-subgroup P, and N = the normalizer in G(k) of a maximal split torus contained in P). In particular, if G is k-anisotropic, then such a Tits system is trivial, i.e., B =G(k) = N.

Andrei Rapinchuk (University of Virginia, Charlottesville) — *On algebraic groups with the same tori*

Let G be an absolutely almost simple simply connected algebraic group over a field K. One defines the genus $\mathbf{gen}_K$(G) to be the collection of K-isomorphism classes of K-forms G' of G that have the same isomorphism classes of maximal K-tori as G. We will discuss conjectures and some recent results about the size of $\mathbf{gen}_K$(G) over finitely generated fields K of good characteristic, focusing primarily on the following two questions:

(1) When does $\mathbf{gen}_K$(G) reduce to a single element?

(2) When is $\mathbf{gen}_K$(G) finite?

This is joint work with V. Chernousov and I. Rapinchuk.

Igor Rapinchuk (Harvard University, Cambridge) — *On the conjecture of Borel and Tits for abstract homomorphisms of algebraic groups*

I will discuss several results on the conjecture of Borel and Tits (1973) on abstract homomorphisms of semisimple algebraic groups. I will also give several applications to the study of character varieties of finitely generated groups.

Donna Testerman (EPFL, Lausanne) — *Overgroups of regular elements in simple algebraic groups*

Let G be a simple algebraic group defined over an algebraically closed field. We study the reductive overgroups of regular elements in G. (Recall that an element x is regular if its centralizer in G has dimension equal to the rank of G.) For the case where x is a regular unipotent element, this question had previously been considered by Saxl and Seitz. Their work naturally leads to the question of whether a reductive subgroup of G containing a regular unipotent element can lie in a proper parabolic subgroup of G. We settle this question for connected reductive subgroups.

In the case where x is a regular semisimple element, we obtain results about maximal closed connected subgroups of G containing x. We also show how one can use the unipotent and semisimple cases to treat the general case.

This is joint work with Alex Zalesski.

Matthias Wendt (Albert-Ludwigs Universität, Freiburg) — *Homology of GL(2) via buildings*

In the talk, I will explain how the action of GL_2 over the function ring of a smooth affine curve on its associated Bruhat-Tits building gives rise to a couple of new computations of group homology of GL_2. These computations exhibit interesting degree four homology classes which cannot be detected on the maximal torus.

Annette Werner (Goethe Universität, Frankfurt) — *Analytic and tropical Grassmannians*

We show that the tropical projective Grassmannian of planes is homeomorphic to a closed subset of the Berkovich analytic Grassmannian by constructing a continuous section of the tropicalization map. Our construction uses the interpretation of the tropical Grassmannian as a space of phylogenetic trees. This is joint work with Mathias Hbich and Maria Angelica Cueto.

Kirill Zainoulline (University of Ottawa) — *Hyperbolic formal group laws, Hecke algebras and generalized Schubert calculus*

In a series of papers Kostant and Kumar introduced and successfully applied the language of nil Hecke and 0-Hecke algebras to describe the T-equivariant cohomology/K-theory of flag varieties. Recently, in the joint projects with Calmés, Savage, Zhong and others we partially extended this approach to the context of arbitrary algebraic oriented cohomology theories (in the sense of Levine-Morel) and formal group laws.

In the present talk we discuss several concrete examples and applications dealing with some special class of elliptic formal group laws (hence, with the associated algebraic oriented elliptic cohomology theories) and show how they are related to classical Iwahori-Hecke algebras, to solutions of the Yang-Baxter equation and Root polynomials.

Changlong Zhong (University of Alberta, Edmonton) — *Formal Demazure algebra and T-equivariant cohomology of flag varieties*

In this talk I will introduce the definition and properties of the formal affine Demazure algebra. This is an algebra constructed using formal group law and root datum. Its dual is thought as the algebraic replacement of the T-equivariant algebraic oriented cohomology of flag varieties.

Xinwen Zhu (Northwestern University, Evanston) — *affine Grassmannians and the geometric Satake in the mixed characteristic*

I will endow the set of lattices in $\mathbf{Q}_p^n$ with a reasonable algebro-geometric structure and will establish the geometric Satake correspondence in this setting. Time permitting, I will also discuss some applications of our theory to the study of Rapoport-Zink spaces.

Maksim Zhykhovich (Johannes-Guntenberg Universität, Mainz) — *Integral Chow motives of projective homogeneous varieties*

Let X be a projective homogeneous variety under the action of a semi-simple algebraic group G. Two decompositions of the integral Chow motive of X are relatively equivalent if they become the same over an algebraic closure of the base field. In this talk I will give a classification of all integral motivic decompositions modulo relative equivalence in terms of reductions modulo torsion primes of G. As an application, I will discuss the Krull-Schmidt principle for integral motives and provide a complete list of generalized Severi-Brauer varieties with indecomposable integral motive. This talk is based on a joint work with Nikita Semenov.