Algebraic Groups and Representations
Lyon
June 2, 2014 to July 11, 2014

Conference: Representations of Algebraic Groups

July 7-11, 2014

Pramod Achar (Louisiana State University, Baton Rouge) — *Modular perverse sheaves on flag varieties II*

This talk is a report on joint work with Simon Riche. (It is a sequel to his talk, but I will try to make it somewhat self-contained.) In characteristic zero, "mixed sheaves" are a powerful tool for studying Koszul phenomena, based on difficult results of algebraic geometry (such as the Weil conjectures). I will review what mixed sheaves are, and what their role in the theory is, following Beilinson-Ginzburg-Soergel and others. But the main focus of the talk will be positive characteristic, where classical mixed methods are largely unavailable. I will explain a proposed new definition of "mixed modular sheaves" that avoids the difficult classical machinery, but seems to have favorable properties. One application of this new framework is the following theorem: In good characteristic, the mixed modular derived category of a flag variety is equivalent to that of the Langlands dual flag variety, via a "Koszul-type" equivalence that swaps parity sheaves with tilting sheaves. If time permits, I will also discuss applications to formality and positivity questions.

Prakash Belkale (University of North Carolina, Chapel Hill) — *Gauss-Manin representations of conformal block local systems*

Conformal blocks, or spaces of generalized theta functions (attached to a group G and representations at a level k), give projective local systems on moduli spaces of curves with marked points. One can ask if they are realizable in geometry, i.e., as local subsystems of suitable Gauss-Manin local systems of cohomology of families of smooth projective varieties.

I will discuss (in genus 0) the proof of Gawedzki et als conjecture that Schechtman-Varchenko forms are square integrable (this was proved first for sl(2) by Ramadas). Together with the flatness results of Schechtman-Varchenko, and the work of Ramadas, one obtains the desired realization and a unitary metric on conformal blocks.

I will also discuss recent results on the image of conformal block local systems in cohomology (joint with S. Mukhopadhyay), and the known results regarding cohomological realizations of the entire space of invariants.

Tristan BOZEC (Université Paris sud) — *Varieties of representations of quivers with loops*

We study generalizations of Lusztig canonical and semicanonical bases, in the case of arbitrary quivers, possibly carrying loops. To that end, we have to build a Hopf algebra generalizing usual quantum groups. We also study Nakajima quiver varieties in the case of arbitrary quivers. We construct some Lagrangian subvarieties, and from their geometric study arises a new combinatorial structure, generalizing Kashiwara crystals.

Paolo Bravi (Università La Sapienza) — *The moduli scheme of affine spherical varieties with a free weight monoid*

In this talk, we consider Alexeev and Brion's moduli scheme M_Γ of affine spherical varieties with weight monoid Γ, under the assumption that Γ is free. We describe the tangent space to M_Γ at its most degenerate point in terms of the combinatorial invariants of spherical varieties and deduce that the irreducible components of M_Γ are affine spaces. This is a joint work with Bart Van Steirteghem.

Peter Fiebig (Universität Erlanger) — *Periodic patterns in the representation theory of affine Kac-Moody algebras*

We will study a certain subcategory of the category $\mathcal{O}$ for an affine Kac-Moody algebras (in positive level) that is governed by periodic polynomials. It can be interpreted as a characteristic zero analogue of the category of G_1T-modules (for big enough characteristics) and hopefully it helps us to understand the critical level category $\mathcal{O}$, which should, by a conjecture of Feigin, Frenkel and Lusztig, be governed by periodic polynomials as well.

Stéphane Gaussent (Université de Saint-Étienne) — *Hecke algebras for Kac-Moody groups over local fields*

Spherical Hecke or Iwahori-Hecke algebras associated to a reductive group over a local field are well known and have a lot of applications in representation theory. Braverman, Kazhdan (and others) have extended their construction to the case of affine Kac-Moody groups. In the talk, we will construct these algebras for any Kac-Moody group over a local field using the hovel (an adapted version of the Bruhat-Tits building). This is a joint work with Nicole Bardy-Panse and Guy Rousseau.

Anthony Henderson (University of Sydney) — *Modular generalized Springer correspondence (parts 1 and 2)*

These talks report on an ongoing project of Pramod Achar, Anthony Henderson, Daniel Juteau and Simon Riche.

We have in mind a future theory of modular character sheaves, which would provide a geometric interpretation of the decomposition matrix of a finite group of Lie type. As a step towards this, we are establishing a modular generalized Springer correspondence for any connected reductive complex group; this has a similar form to Lusztig's generalized Springer correspondence, but involves representations and perverse sheaves over a field of positive characteristic. The change to modular coefficients creates many difficulties and new features.

The first talk will explain the motivation and our general results about parabolic induction and restriction of modular perverse sheaves on the nilpotent cone. The simple perverse sheaves are partitioned into induction series, attached to cuspidal pairs for Levi subgroups. The elements of each series are in bijection with the modular irreducible representations of the relative Weyl group. Compared to Lusztig's situation, there are more cuspidal pairs; in particular, the Levi subgroups that support them do not have to be self-opposed, and the relative Weyl groups do not have to be Coxeter groups.

The second talk will introduce further general results, and summarize the state of our knowledge for the various types of quasi-simple group. In type A, the classification of cuspidal pairs and the explicit determination of each series is complete, and is appropriately reminiscent of known results on unipotent Brauer characters for the finite general linear group. In other classical types, the classification of cuspidal pairs is complete, but we have determined the induction series only in characteristic 2. In the exceptional types, several indeterminacies remain.

Daniel Juteau (Université de Caen) — *Modular generalized Springer correspondence (parts 1 and 2)*

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Allen Knutson (Cornell University, Ithaca) — *Asymptotics of branching to symmetric subgroups*

Let K be a connected symmetric subgroup of a complex Lie group G, so K acts with finitely many orbits on G/B. We consider the question of decomposing finite-dimensional irreps V_λ of G under K, in the asymptotic limit where λ is replaced by $N\lambda$, and give a manifestly positive formula for the leading term: it is the volume of a polyhedral complex, with polyhedra indexed by chains in the K\G/B poset. In the case G = K×K, we can recover an asymptotic form of Littelmann's tensor product formula (as well as more symmetric formulae).

More generally, we define a K-Demazure module as the space of sections of dominant line bundle over a K-orbit closure, and prove an asymptotic formula for these decompositions, by induction over K\G/B.

Shrawan Kumar (University of North Carolina, Chapel Hill) — *A study of saturated tensor cone for symmetrizable Kac-Moody Lie algebras*

This is a joing work with Merrick Brown. Let $\mathfrak{g}$ be a symmetrizable Kac-Moody Lie algebra with the standard Cartan subalgebra $\mathfrak{h}$ and the Weyl group W. Let P_+ be the set of dominant integral weights. For $\lambda \in \mathrm{P}_+$, let $\mathrm{L}(\lambda)$ be the integrable, highest weight (irreducible) representation of $\mathfrak{g}$ with highest weight λ. For a positive integer s, define the *saturated tensor semigroup* as

$$\Gamma_s = \{(\lambda, ..., \lambda, \mu) \in P_+^{s+1} \mid \exists N \geqslant 1 \text{ with } \mathrm{L}(N\mu) \subset \mathrm{L}(Nlambda) \otimes \cdots \otimes \mathrm{L}(N\lambda)\}.$$

The aim of this work is to begin a systematic study of Γ_s in the infinite dimensional symmetrizable Kac-Moody case. In this work, we produce a set of necessary inequalities satisfied by Γ_s. These inequalities are indexed by products in $\mathrm{H}(\mathrm{G}^{\mathrm{min}}/\mathrm{B}, \mathbf{Z})$ for B the standard Borel subgroup, where $\mathrm{G}^{\mathrm{min}}$ is the minimal Kac-Moody group with Lie algebra $\mathfrak{g}$. The proof relies on the Kac-Moody analogue of the Borel-Weil theorem and Geometric Invariant Theory (specifically the Hilbert-Mumford index). In the case that $\mathfrak{g}$ is affine of rank 2, we show that these inequalities are necessary and sufficient. We further prove that any integer $d > 0$ is a saturation factor for $\mathrm{A}_1(1)$ and 4 is a saturation factor for $\mathrm{A}_2(2)$.

Martina Lanini (Universität Erlangen-Nürenberg) — *Degenerate flags and Schubert varieties*

Introduced in 2010 by E. Feigin, degenerate flag varieties are flat degenerations of flag manifolds. In the type A and C, they share many properties with Schubert varieties, such as being normal locally complete intersections, in general singular with rational singularities. In this talk I will discuss joint work with Cerulli Irelli, where we prove a surprising fact about degenerate flags.

George Lusztig (MIT, Cambridge) — *Some applications of the asymptotic Hecke algebra*

Ivan Mirkovic (University of Massachusetts, Amherst) —

Anne Moreau (Université de Poitiers) — *Jet schemes of the nilpotent orbit closures in a simple Lie algebra*

By a result of Eisenbud and Frenkel, all jet schemes of the nilpotent cone of a complex simple Lie algebra are irreducible. In this talk, we will focus on the jet schemes of the other nilpotent closures.

This is a joint work with Rupert Yu, and it is still in progress.

Daniel K. Nakano (University of Georgia, Athens) — *Tensor triangular geometry and classical Lie superalgebras*

Tensor triangular geometry as introduced by Balmer is a powerful idea which can be used to extract the ambient geometry from a given tensor triangulated category. In this talk I will first present a general setting for a compactly generated tensor triangulated category which enables one to classify thick tensor ideals and the spectrum Spc. For a classical Lie superalgebra $\mathfrak{g}$, I will later show how to construct a Zariski space from a detecting subalgebra f and demonstrate that this topological space governs the tensor triangular geometry for the category of finite dimensional $\mathfrak{g}$-modules which are semisimple over the reductive Lie algebra $\mathfrak{g}_0$. Concrete realizations will be provided for the Lie superalgebra $\mathfrak{gl}(m|n)$. These results represent joint work with B. Boe and J. Kujawa.

Clélia Pech (King's College London) — *On mirror symmetry for cominuscule homogeneous spaces*

In this talk, I will explain how to construct mirrors ('Landau-Ginzburg models') for some cominuscule homogeneous spaces, including Lagrangian Grassmannians and quadrics. These mirrors stem from a general Lie-theoretic construction for homogeneous spaces by K. Rietsch. Here I will give explicit expressions for the mirrors, in "natural" coordinates related to the cohomology of the variety. I will also explain the cluster algebra structure which exists on the mirrors, and give applications to quantum cohomology. This is joint work with respectively K. Rietsch, and K. Rietsch and L. Williams.

Nicolas Perrin (Heinrich Heine Universität, Düsseldorf) — *Split subvarieties of group embeddings*

Simon Riche (Université de Clermont-Ferrand) — *Modular perverse sheaves on flag varieties I*

I will report on a joint work with Pramod Achar which aims to understand the category of Bruhat-constructible perverse sheaves on the flag variety of a reductive algebraic group, with coefficients in a field of positive characteristic. In this first part I will explain a relation between tilting perverse sheaves on a flag variety and parity sheaves on the Langlands dual flag variety, and some applications to the computation of multiplicities of simple perverse sheaves in standard perverse sheaves, and to a geometric description of Soergel's modular category $\mathcal{O}$.

Steven Sam (University of California, Berkeley) — *Representations of Lie superalgebras and determinantal varieties*

Several families of classical Lie superalgebras (e.g., general linear, orthosymplectic, periplectic) have consistent $\mathbf{Z}$-gradings such that the positive part is the homotopy Lie algebra of a commutative k-algebra A. This induces an action of its positive part on $\mathrm{Ext}^*_{\mathrm{A}}(\mathrm{M},k)$. In this talk I will explain some examples coming from determinantal varieties where the action of the positive part extends to the whole Lie superalgebra to give an irreducible representation. This might be thought of as a Koszul dual analogue of the representations of Lie algebras coming from Howe duality in classical invariant theory.

Olivier Schiffman (Université Paris 11 Sud, Orsay) — *Indecomposable vector bundles and stable Higgs bundles on curves*

We will give an formula for the number of indecomposable vector bundles of fixed rank and degree over a smooth projective curve X of genus g defined over a finite field $\mathbf{F}_q$. This formula is an explicit polynomial in the Weil numbers $(\sigma_1, \ldots, \sigma_{2g})$ of the curve; for instance in rank one it is the well-known formula $\prod_i (1 - \sigma_i)$ for the number of points in the Jacobian of X.

We then prove that (when the rank and degree are coprime) the number of indecomposable vector bundles is equal (up to a power of q) to the number of stable Higgs bundles over X. This entails a closed formula for the Poincare polynomial of the moduli space of stable Higgs bundles.

Ben Webster (University of Virginia, Charlottesville) — *TBA*