Possibly non-affine algebraic groups

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The course will consist of 4 lectures of 90 min each on the structure of algebraic groups (possibly non-linear).

A fundamental result of Chevalley asserts that every connected algebraic group over a perfect field is an extension of an abelian variety by a connected linear algebraic group, in a unique way. This result fails over any imperfect field, but can be extended to that setting by considering subgroup schemes (Raynaud) or pseudo-abelian varieties (Totaro).

The course will present Chevalley's theorem and related structure results over an algebraically closed field, and then discuss the case of an arbitrary base field.

Prerequisites

Basic notions of algebraic geometry, e.g., Chapters 1 and 2 of Hartshorne's book. Some familiarity with abelian varieties and/or the structure of linear algebraic groups over algebraically closed fields may be helpful, but not compulsory.

Some references

Brion, M., Samuel, P., Uma, V. — *Lectures on the structure of algebraic groups and geometric applications*, CMI Lecture Series in Mathematics 1, Hindustan Book Agency, 2013.
Available at `http://www-fourier.ujf-grenoble.fr/ mbrion/chennai.pdf`

Conrad, B. — A modern proof of Chevalley's theorem on algebraic groups, *J. Ramanujan Math. Soc.* 17 (2002), 1-18.

Rosenlicht, M. — Some basic theorems on algebraic groups, *Amer. J. Math.* 78 (1956), 401-443.

Totaro, B. — Pseudo-abelian varieties, *Ann. scient. Ec. Norm Sup.* 46 (2013), 693-701.