Path Model

Stéphane Gaussent

This course will focus on the use of the Littelmann's path model in the proof of the following "saturation" theorem given by Kapovich, Leeb and Millson.

THEOREM — *Let G be a semi-simple complex group and T a maximal torus in G. Let k be the least common multiple of the coefficients of the biggest coroot of (G,T). Let a, b and c be dominant weights (associated to a choice of a Borel subgroup B containing T) such that $a+b+c$ is a root. If there exists an integer N such that $V(Nc)$ is a subrepresentation of the tensor product of $V(Na)$ and $V(Nb)$, then $V(k^2c)$ is a subrepresentation of the tensor product of $V(k^2a)$ and $V(k^2b)$.*

The other main ingredient of the proof is the Bruhat-Tits building associated to G and the field of Laurent series with complex coefficients.

In the case of a group of type A ; k = 1, and we get another proof of a theorem first established by Knutson and Tao, using the honeycomb model, and by Derksen and Weyman with the help of quivers.

References

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