

Affine groups in positive characteristic

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The course will present foundational material on affine algebraic groups defined over an arbitrary base field k , including the case of positive characteristic. It comes mainly from the SGA3 seminar, in particular exposés VIA and XII. We shall partially follow also the appendices of the "Pseudo-reductive groups" book, linking then the course will that of Brian Conrad (of the second week).

An affine algebraic k -group is a subgroup of a linear group $GL(n)$. Basic examples are classical groups like the orthogonal group, tori (occurring as diagonal part) and unipotent subgroups as the subgroup of strictly upper triangular matrices. We shall discuss other explicit examples and explain the relevancy of the scheme theoretical viewpoint on this topic. The rough plan is the following :

1. Descent and representability, Weil restriction, smooth algebraic groups.
2. k -orbits, homogeneous spaces, unipotent and solvable groups.
3. Unipotent radicals, maximal tori, Levi subgroups and structure theorems.

References

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WATERHOUSE, W. — *Introduction to affine group schemes*, Springer (1979).