Geometric character theory of $GL_n(\mathbf{F}_q)$

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The character table of the finite group $GL_n(\mathbf{F}_q)$ was completed in 1955 by Green using combinatorial methods. These methods showed their limit for the study of the character theory of other finite groups of Lie type like $SL_n(\mathbf{F}_q)$.

From the seventies to the end of the eighties, Deligne, Kazhdan, Springer and mainly Lusztig developped incredible geometric tools to investigate the character theory of any finite groups of Lie type. These methods, based on l-adic cohomological methods and on the theory of perverse sheaves, were extended in many other areas of representation theory.

In this course I will introduce some of these tools through the character theory of $GL_n(\mathbf{F}_q)$. I will in particular focus on the construction of the unipotent characters of $GL_n(\mathbf{F}_q)$ using the theory of perverse sheaves.

The rough plan is as follows.

1. Combinatorial construction of the unipotent characters of $GL_n(\mathbf{F}_q)$.
2. A pragmatic approach to the theory of perverse sheaves.
3. Geometric realization of the unipotent characters of $GL_n(\mathbf{F}_q)$.

Required background — Basic knowledge on linear algebraic groups and representations of finite groups (ex : classical induction).

References

MACDONALD, I.G. — *Symmetric Functions and Hall Polynomials*, Oxford mathematical monographs (1995). [Chapter IV]

LUSZTIG, G. — Green polynomials and Singularities of Unipotent Classes, *Advances in Mathematics* (1981).

HENDERSON, A. — Fourier transform, Parabolic Induction and Nilpotent Orbits, *Transformation Groups* (2001).