Bruhat-Tits theory

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The course will consists of 4 lectures of 90 min each on the Bruhat-Tits theory of reductive groups over local fields.

Familiarity with the structure of reductive groups over arbitrary fields (Borel-Tits theory) could help, but a summary of the structure of rational points will be given during the first lecture. It will be formulated in combinatorial terms ; in other words, we will explain how the structure of Tits systems can sum up the main results of Borel-Tits theory.

A substantial part of Bruhat-Tits theory can then be seen as a refinement of the latter combinatorial theory taking into account the fact the ground field carries a nice filtration. The geometric counterpart to this is the existence of a highly transitive action of the rational points of the group on a building, i.e. a non-positively curved cell complex with strong symmetry properties. The other half of the theory deals with the fact that some compact open subgroups can be seen as point of models of the group over the valuation ring of the (local) ground field.

Here are the topics (possibly up yo minor permutation) :

1. Euclidean buildings, metric properties
2. Borel-Tits theory, structure of rational points, valuations on root group data
3. The example of the group $\mathrm{GL}(n)$
4. Integral structures
5. Decompositions (Cartan, Iwasawa) and applications
6. The idea of the descent ; examples of unitary groups with three variables

Some references

Abramenko, P. and Brown, K. — *Buildings : theory and applications*, Graduate Texts in Mathematics, vol. 248, Springer (2008).

Landvogt, E. — *A compactification of the Bruhat-Tits building*, Lecture Notes in Mathematics 1619, Springer (1996).

RÉMY, B., THUILLIER, A. and WERNER, A. — Bruhat-Tits buildings and analytic geometry. *Proceedings of the Paris summer school "Berkovich spaces" (July 2010).*
Available at `http://math.univ-lyon1.fr/homes-www/remy/maths.html`

SPRINGER, T.A. — *Linear algebraic groups*, 2nd edition, Birkhauser (1998).

TITS, J. — Reductive groups over local fields, *Proceedings on the Corvallis conference (1977)*, American Math. Soc. (1979).