

## Kazhdan-Lusztig theory

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Hecke algebras are certain  $q$ -deformation of the group algebra of Coxeter groups. In 1979 Kazhdan and Lusztig introduced a distinguished basis of the Hecke algebra. The coefficients of the transfer matrix between this basis and the standard basis of the Hecke algebra give rise to the Kazhdan-Lusztig polynomials. These polynomials enjoy beautiful categorical interpretations : in the case of finite or affine Weyl groups they are related to intersection cohomology of Schubert varieties and to character formulae for highest weight simple representations of semi-simple Lie algebras. A categorical interpretation for all Coxeter groups has also been given by the work of Soergel and Elias-Williamson, as will be discussed during the 5th week of the program.

In this course, we will focus on the classical aspects of the theory, and will cover the following topics :

1. Hecke algebras, their canonical bases and Kazhdan-Lusztig polynomials.
2. Intersection cohomology of Schubert varieties.
3. Kazhdan-Lusztig conjecture on multiplicities of Verma modules in the category  $\mathcal{O}$  of semisimple Lie algebras.

### *References*

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